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ABSTRACTS

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Contents

Plenary lectures

Loci of centers in pencils of triangles in the isotropic plane

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In this presentation we study loci of some centers in pencils of triangles in the isotropic plane, a real projective plane where the metric is induced by an absolute figure (F, f) consisting of a real line f and a real point F incident to it.

First we consider a triangle pencil consisting of the triangles that have the same circumcircle, i.e. two vertices are fixed, and the third vertix runs along the circumcircle. We study the curves of centroids, symmedian points, Gergonne points and Brocard points for such pencil of triangles, and show that the curve of centroids is a circle, the curve of symmedian points is an ellipse, the curve of Gergonne points is a 2-circular and rational quartic that touches the absolute line at the absolute point, while the curves of Brocard points are entirely circular quartics. We also study loci of centers of tangential triangles of such triangle pencil.

Then, we consider a triangle pencil consisting of those triangles that have two fixed vertices, while the third vertex is moving along a line. We prove that the curve of centroids of all triangles from the pencil is a line, the curve of symmedian points is an ellipse, the curve of Gergonne points is a rational cubic, the curve of the Feuerbach points is a 2-circular and rational cubic having an ordinary double point at the absolute point, while the curves of Brocard points are entirely circular and rational quartics.

Any triangle in the isotropic plane has a circumcircle c and an incircle i. It turns out that there are infinitely many triangles with the same circumcircle c and incircle i. This one-parameter family of triangles is called a poristic system of triangles. We prove that all triangles in the poristic system share the centroid and the Feuerbach point. The symmedian point and the Gergonne point of the triangle $P_1P_2P_3$ move on the lines while the triangle traverses the poristic family. The Steiner point of $P_1P_2P_3$ traces a circle, and the Brocard points of $P_1P_2P_3$ trace a quartic curve (Figure 1).

Key words: isotropic plane, pencil of triangles, centroid, Gergonne point, symmedian point, Brocard points, Feuerbach point, Steiner point, Poncelet porism

MSC 2020: 51N25

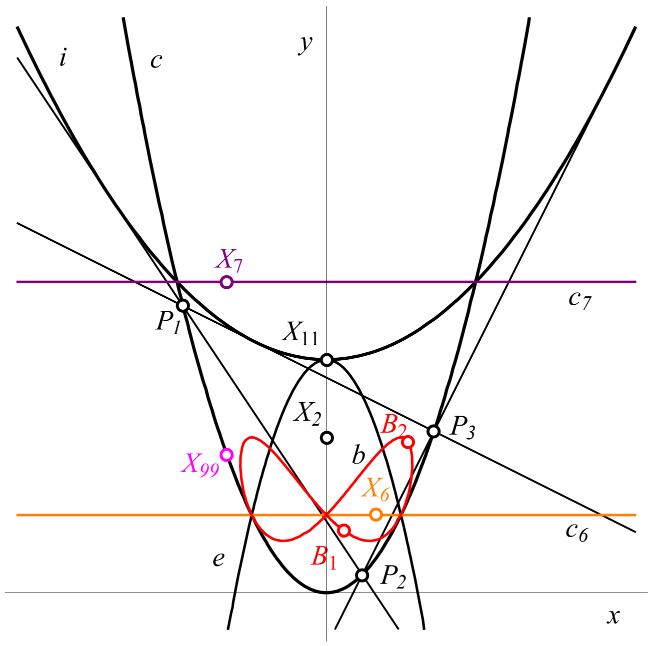

Figure 1.

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The role of Geometry in Architecture - What make architecture beautiful?

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What is the difference between good and bad architecture? Is it form, material, arrangement of rooms, implementation of details? Is it a subjective assessment of what is good and what is not? Regardless of which of the above statements we agree with or not, the fact is that there are good buildings and equally bad buildings. The good ones are published into professional magazines, books, tourist guides, are under monument protection or even find themselves on the UNESCO list. The problem of quality architecture was dealt with already in antiquity. Vitruvius left us the postulates that say that good architecture must be solid (durable), useful (comfortable) and beautiful. If there are still measurable criteria for the first two, there is a bigger problem with the evaluation of beauty. Researchers who deal with the analysis of buildings say that the composition and arrangement of the components is what makes a building beautiful. Ancient planners based these canons of beauty on geometric patterns and proportions. The use of basic geometric figures and bodies and the ratio of small whole numbers was the compositional key to achieving beauty. Two more compositional methods are associated with this: the use of the ratio between the circle's circumference and diameter (the number π) and the golden ratio. The harmony of the composition and comfort were achieved by using an anthropometric measuring system, the basis of which is the human body. The anthropometric measurement system was in effect until the introduction of the metric measurement system. The latter was first introduced in France in 1791 and then spread to most countries of the world. The metric measurement system is based on a cosmic measurement, and is not good for composability like the anthropometric system. It is true, however, that the metric measurement system enabled unification of measurements in a wider area and standardization in construction.

The transition from one system to another meant finding how to combine the advantages of one and the other system, or searching for a system that would combine both measurement systems. Le Corbusier came closest to this with his Modulor. He used only this in planning his architecture. On the other hand, we have Gaudí, who considered himself a geometer and explored new geometric forms. He achieved the beauty of his architecture by imitating patterns from nature. He gave natural forms the shape of geometric elements (bodies, curves...) and then used them in his works. It is a wealth of elements of various shapes (curved surfaces, helicoids, paraboloids...), which are better understood only today with the help of various computer simulations. Plečnik is similar to him, who relied on ancient and Renaissance traditions and the use of old compositional principles based on the use of correct geometric forms and proportions. He used these mainly in sacral architecture, which he believed should be the best. According to some researchers of architectural principles, many top architects also used gematria to design the composition. It is a principle that converts text messages into numbers that serve as the basis of the composition. According to Kurent, it is an old compositional principle that was part of a hidden science and that Plečnik was also supposed to know and use. The claim is hard to believe given the various interpretations gematria offers. Today's architecture tries to find its artistic expression in simplified forms and following some "natural" forms. Today's way of life forces them to do so, where speed is an important factor in construction. In the foreground is sustainability and social engagement. Achieving beauty in the classical sense is less important. Architects of the present time want to achieve beauty by using the so-called architecture, stripped of all "cover". A decorative element, if it is not implemented as a detail or part of the construction, is redundant. This is shown by an analysis of selected works by Pritzker laureates. The prize is considered the "Nobel Prize" of architecture and is awarded once a year. Most of the winners of recent years mostly use flat surfaces and rectangles in their designs. The reasoning of the jury also confirms this. Socially engaged architecture is preferred. But there are also exceptions. Perhaps the most exposed are Otto Frei, who experimented with new forms, and Zaha Hadid, who relied on the so-called natural forms, which are essentially demanding geometric shapes. It is characteristic of both of them that they had a very good spatial perception. The latter is important even in the age of computers, but unfortunately, with the changed way of life, it is slowly being lost or its level is decreasing. But it is possible to acquire it, to increase it through exercises and tasks that are close to descriptive geometry.

Key words: geometry, architecture, composition

MSC 2020: 00A06, 00A66, 00A67, 91C05

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Archimedean solids known in the 15th and 16th centuries

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During the 15th and 16th centuries, a few artists and mathematicians studied solid bodies whose vertices *touch a hollow sphere*, to use Albrecht Dürer's words, unaware that Archimedes had described thirteen of these centuries earlier. Johannes Kepler named them the *Archimedêa Corpora* in the "Harmonices Mundi", which was published in 1619 and marks a significant milestone in the history of polyhedra. Piero della Francesca had described six of these solid bodies, Luca Pacioli described two more, and Leonardo da Vinci described another. Albrecht Dürer described two others, while Augustin Hirschvogel described one more. Daniele Barbaro has been identified [1] as the author who described more Archimedean solids before Kepler, including one previously unknown. It is, however, important to note that an Anonymous Author had described the thirteen Archimedean solids long before Kepler and even Barbaro. The author discusses her findings from a comprehensive comparative analysis of the description of the Archimedean solids in the 15th and 16th centuries, encompassing their early discovery and historical importance in the context of the history of geometry.

Key words: geometry, polyhedra, history of science, Archimedean solids

MSC 2020: 51-03, 01A40, 00A66

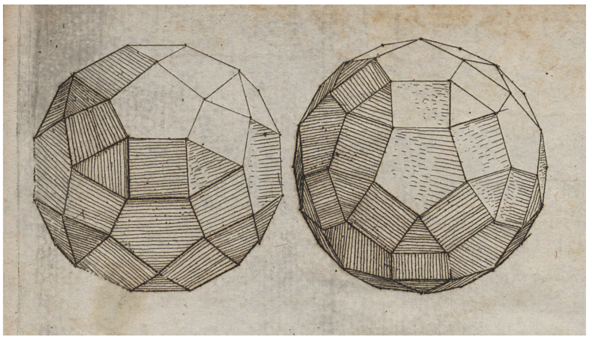

Figure 1: Detail of Plate A VII of Augustin Hirschvogel's *Geometria* [2]

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Contributed talks

Generative geometric arts revisited

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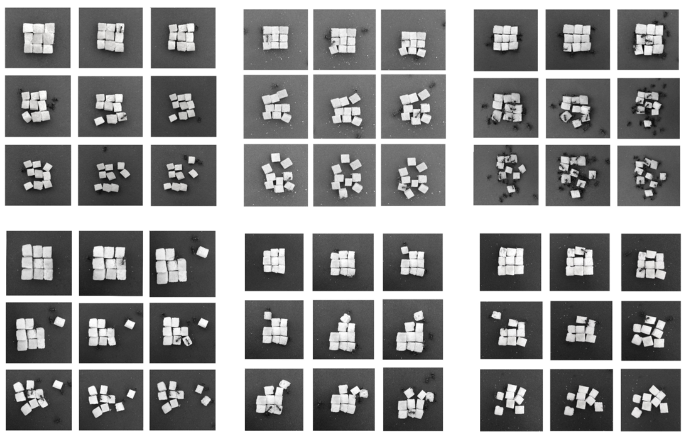
The truly interdisciplinary research was based on the generative, algorithmic art movement that started at the beginning of the last century, whose best-known Hungarian representative was Vera Molnár. She created artworks by modifying an initially well-ordered geometric structure with the computer's random generator. Inspired by her works, we tried to create similar shapes with the help of natural art tools, live ants, from which photographs, collages and videos were born. The scientifically and artistically relevant goal of the research was to find out how different or similar the computer-generated pseudorandomness is to naturally occurring randomness. We also discuss the statistical and educational aspects of this field.



Key words: generative geometric art, Vera Molnár, randomness

MSC 2010: 97G20, 97G50, 97G80, 62H35

Application Mathtvz

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In this talk the process of developing MathTVZ web application will be described. The application helps first year students of informatics in their process of understanding and learning the teaching material of the course Mathematics I at the University of Applied Sciences in Zagreb. The first part of the presentation describes all technologies and programming languages used for development. The central part of the presentation describes the functionality and programming of the application itself, and at the end lists, the testing methods, application development results and upgrade options.

Key words: Mathematics, application

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Experiments in direction field synthesis using Wave Function Collapse

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Direction fields are used in many different areas, ranging from Computer Graphics to Geometry Processing. Among the requirements that such fields have to meet are smoothness and adherence to alignment constraints.

While common methods to generate smooth direction fields involve numerical optimization routines, we describe the generation of direction fields using *Wave Function Collapse* (WFC), an algorithm that is commonly used for procedural generation of images and geometry. It assigns states to sample points via constraint solving. WFC was initially designed to work only with regular grid structures containing the sample points. We extended it to work also with general graph structures which allows for arbitrary spatial sampling, for example triangulated 3D surfaces. We employ an iterative backtracking procedure that clusters the generated singularities based on density and restarts constraint solving for sample points inside these clusters. This approach drastically reduces the amount of generated singularities.

Solutions found using our approach are always guaranteed to adhere to the input constraints but are not necessarily globally optimal, i.e. containing a minimal amount of singularities, due to the non-deterministic nature of the algorithm. However, its main strength is that it can find a variety of direction fields that satisfy a predefined set of constraints which can be used for artistic purposes, like procedural generation, where optimality is secondary.

Key words: procedural generation, direction fields, Wave Function Collapse, constraint solving

MSC 2020: 05C15, 58K45

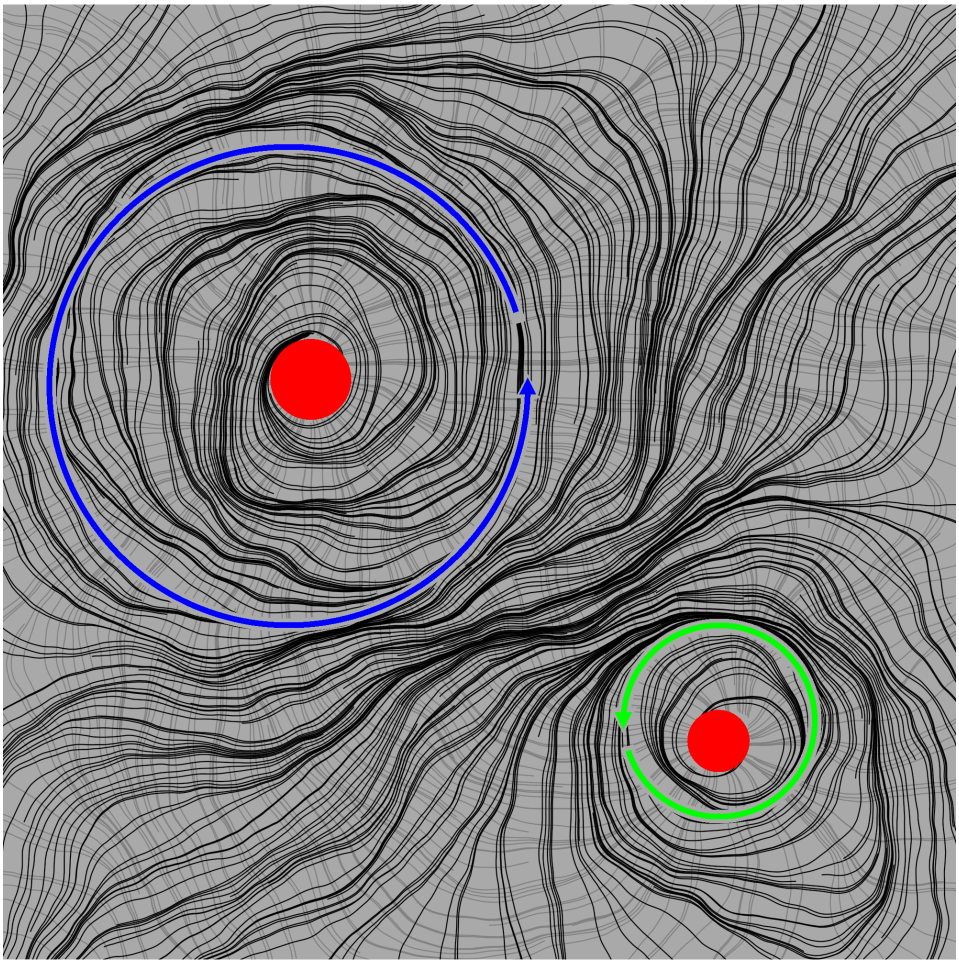

Figure 1: Streamlines traced from a 2D direction field generated by our algorithm. Two circular constraints (blue and green arcs) of opposing directions were placed as initial input. Red circles indicate the centers of the final singularity clusters determined by our system after multiple backtracking iterations.

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O.P. before O.P. in Philibert de l'Orme: the early bases of a new language

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After the *pictorial approach* to Geometry and Graphics culminating in the Renaissance perspective during the fifteenth century, in the sixteenth century architects began to focus on a *technical approach* that could help architectural design and construction.

While painters aimed at obtaining an appropriate *visual control*, architects were aiming at obtaining an appropriate *metric control* through the image.

As the main implication, they had to create a new graphic language.

The french architect Philibert de l'Orme (1514-1570) is one of the most relevant initiators of this new course (Fig. 1), which will have led to the establishment of the Monge's projections about two centuries later.

We would like to propose a synthetic retrospect on the early stirrings of this new language, which required a totally new way of thinking in the use of Geometry and Graphics, to correctly managing the transcription of three-dimensional metric information on the bi-dimensional surface of the paper, and to finally converting projects into construction.

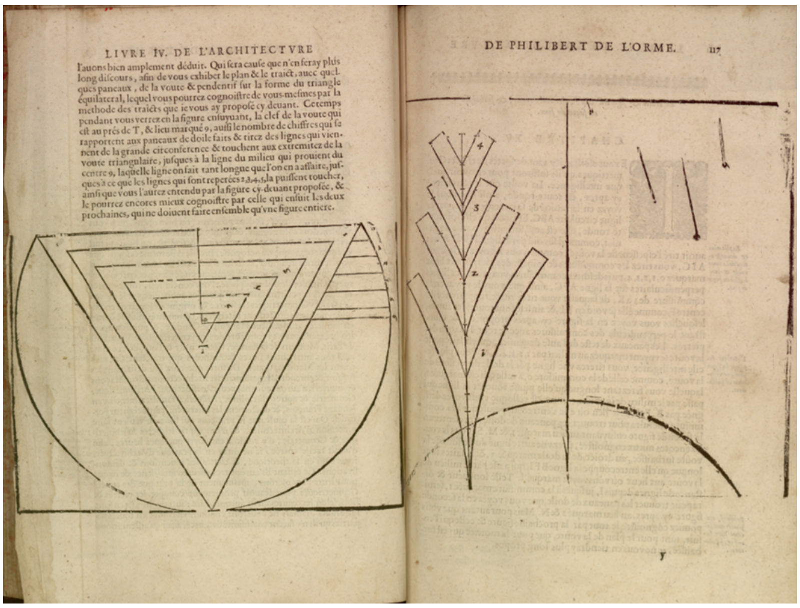

Figure 1: Veil vault on a triangular base. Diagram by Philibert de l'Orme, 1648 [1].

Key words: Philibert de l'Orme, architectural stereotomy, geometry and graphics

MSC 2020: 00A05, 00A66, 51N05, 01A05, 97U99

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Worpitzky numbers and acyclic graph

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For the nonnegative integers n, k, the Worpitzky numbers $W_{n,k}$ are defined by recursive relation $W_{n,k} = (k+1)W_{n-1,k} + kW_{n-1,k-1}$ and the initial values $W_{n,0} = 1$ and $W_{1,1} = 1$.

In this paper we present a combinatorial interpretation of these numbers by means of planar graphs. We also give an interpretation of set partitions within the planar graph. It has been shown that the path enumeration gives identities involving Worpitzky numbers.

Key words: Worpitzky numbers, planar graph, set partitions

Assessment in mathematics in the age of AI: Shifting the focus from problem solving to problem posing

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The rapid development of Artificial Intelligence (AI) asks for redefining assessment in mathematics and exploring its impact on traditional assessment methods and intended learning outcomes [1]. AI-powered tools offer immense potential for formative and summative assessment, including automated grading, adaptive testing, and personalized feedback [4]. However, these advancements raise ethical concerns, particularly regarding student cheating and biases in AI algorithms and compliance with EU AI Act.

Flipped classroom approaches and problem- and project-based learning (PBL) should be employed more extensively as they bust students self-regulated learning. In PBL, it is crucial to emphasize problem-posing alongside problem-solving in mathematics education [6]. Problem posing foster creativity, critical thinking, and a deeper understanding of mathematics. Assessing student-generated problems can provide valuable insights into their understanding of mathematical concepts and their ability to apply knowledge in novel situations [5]. AI can generate problems tailored to individual student needs, provide feedback on student-created problems, and even assess their quality and complexity.

Despite these challenges, computer-assisted assessment and machine learning algorithms offer opportunities for fairer, more valid, and reliable assessment, as well as deeper insights into student learning and individualized instruction [2], [3]. We must develop a vision for how AI can enhance assessment practices in mathematics to better support student learning and achievement, while also determining relevant mathematical content and teaching/learning methods for higher education [4].

Key words: mathematical education, Artificial Intelligence, assessment, project-based learning

MSC 2020: 97M10

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Geometric graphs on fullerene spectra

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A fullerene graph is a 3-regular, 3-connected, plane graph with only pentagonal and hexagonal faces. Its spectrum is the spectrum of its adjacency matrix. Given a graph F_n on n vertices, its spectrum can be considered as a point in the n-dimensional space. For a given number of vertices n, we compute spectra of all fullerene isomers on n vertices. On the resulting n-dimensional point set we build a family of geometric graphs depending on a positive parameter d, where, for a given value of d, the corresponding graph $G(n, d)$ is obtained by connecting all pairs of points whose distance does not exceed d. We analyze the properties of such geometric graphs, in particular, the evolution of their number of edges and connected clusters with increasing values of d.

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Minimal invariant and minimal totally real submanifolds in Sol_1^4

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We present minimal invariant and minimal totally real submanifolds in the 4-dimensional homogeneous solvable Lie group Sol_1^4 equipped with standard globally conformal Kähler structure. We prove that the only minimal invariant surfaces of Sol_1^4 are totally geodesic hyperbolic planes. Also, we classified the minimal totally real submanifolds with tangent or normal Lee vector field.

Key words: minimal submanifold, LCK manifold, Sol_1^4 space

MSC 2020: 53C15, 53C25, 53C30, 53C55

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On geometrical choices in programmatic implementation of Robin Hood numerical method

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A paper explaining novel BEM-like numerical method for solving electrostatic problems called Robin Hood method was published in 2004 [1]. The method utilized a principle of iterative transfer of the electric charge, e.g. non-local charge transfer from the place of currently highest potential to the place of currently smallest potential for insulated conducting objects (figuratively taking from the rich and giving to the poor [2]). Due to its algorithm, the implementation of the method offered fast convergence and linear scaling of the computer memory with the number of geometrical elements (triangles).

After two decades, we are now revisiting the Robin Hood method for the possibility of transferring it to other applications and fields (namely acoustics). As a starting point, the electrostatic algorithm was reimplemented from scratch with modern programming and research tools. Several considerations were again revisited: the choice of triangulation, the separation of the right-angled triangle as well as recursive calls of the electrostatic potential calculation. In this presentation, we will discuss these findings and give outlook for future work.

Key words: numerical method, electrostatics, triangulation, recursion

MSC 2020: 78M15, 68W40

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Some examples for a contemporary geometry education of engineers & architects

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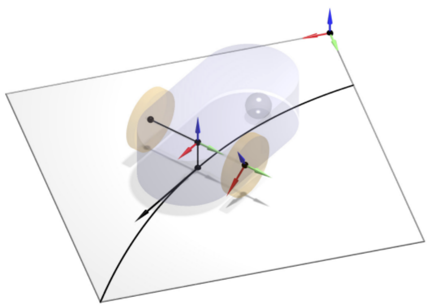
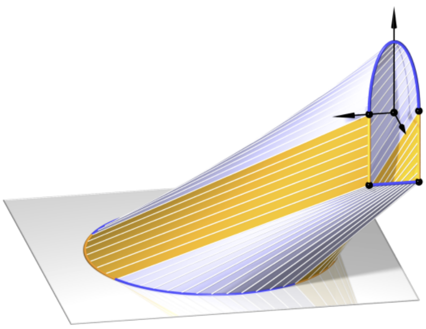
Recently, we observe decreasing geometry skills of first-year students. In our opinion, there are several reasons for that: On the one hand (parts of) geometrical contents have been removed from the high-school curricula. For instance, in maths courses vector analysis is currently taught in dimension 2, only. On the other hand, the percentage of high-school graduates with a solid pre-education in (descriptive) geometry drops from year to year.

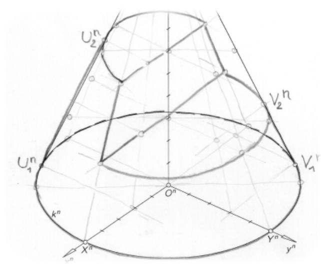
What can we do to compensate this lack of geometry skills? Our approach is to motivate the students in an application-oriented manner: They will rather be willing to learn some geometry if a task is associated with a real problem occurring in their particular discipline. We give some examples for that in our presentation.

Key words: geometric education

MSC 2020: 97D40, 97G70, 97G80







Fakes, illusions, and extraterrestrials

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"Who knows nothing must believe everything." This proverb is from Marie von Ebner-Eschenbach, but it could also serve as the motto for this lecture. Why didn't it take aliens to create the vast and impressive geoglyphs of Nazca in the desert sand (Fig. 1, 2)? Why do aliens always need tractor tracks in the crop fields to leave us their often beautiful geometric patterns (Fig. 3, 4)? How was an android able to win against the best chess players at the end of the 18th century? Why have we been able to measure the constantly changing distance to the moon to within a centimeter since July 21, 1969? These and other questions are resolved through geometric considerations and illustrated with computer animations (see also [1], [2]).

Key words: geometric analysis, historical geoglyphs, crop circles, perspective distortion

MSC 2020: 51Nxx, 68Uxx, 00A69

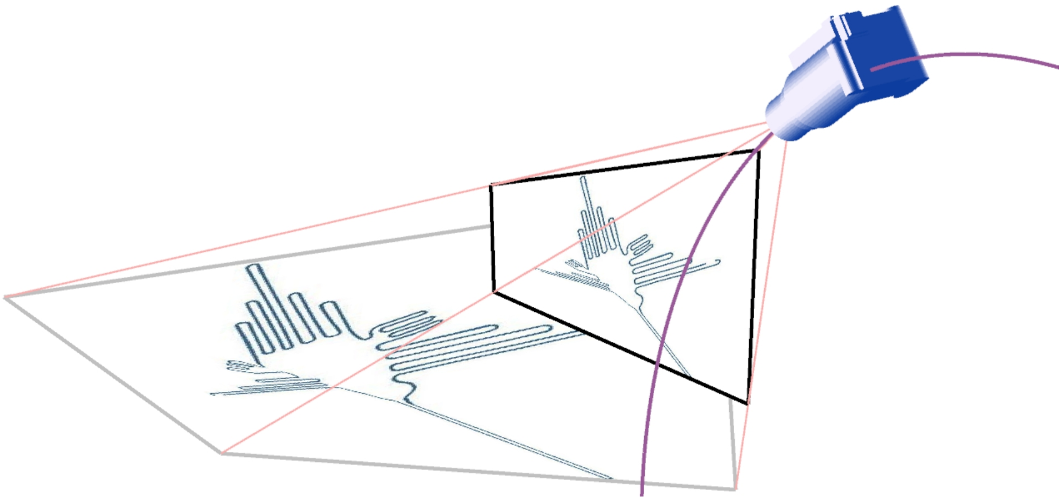

Figure 1: Extremely large geoglyphs in the desert of Peru, created by the Nazcas in pre-Columbian times. From the airplane, they appear undistorted from specific positions.

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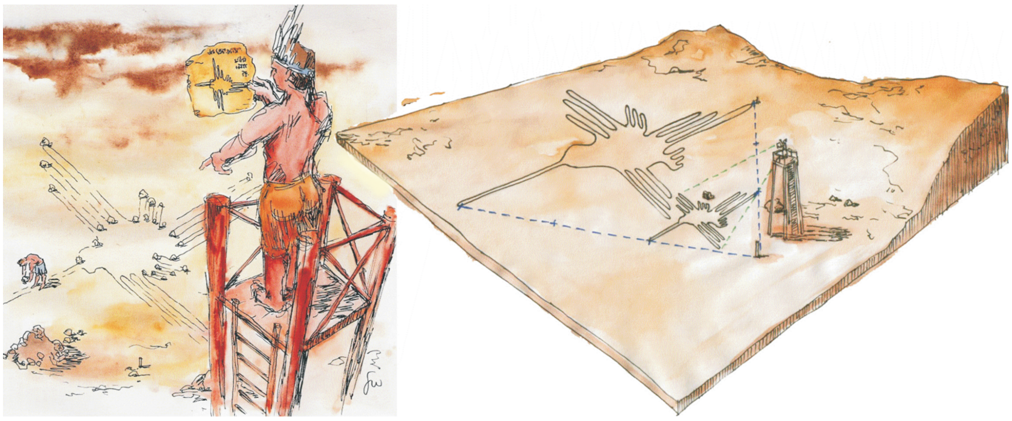

Figure 2: The vast geoglyphs could have been created by scaling up smaller images, as the artist Stefan Wirnsperger has illustrated for the author. This process automatically results in perspective distortions.

Figure 3: The beings that created the impressive, vast image in the crop field seem to have been familiar with subtle geometric constructions – in this specific case, an inversion of circle patterns. (Photo on the left: `https://en.wikipedia.org/wiki/Crop_circle`)

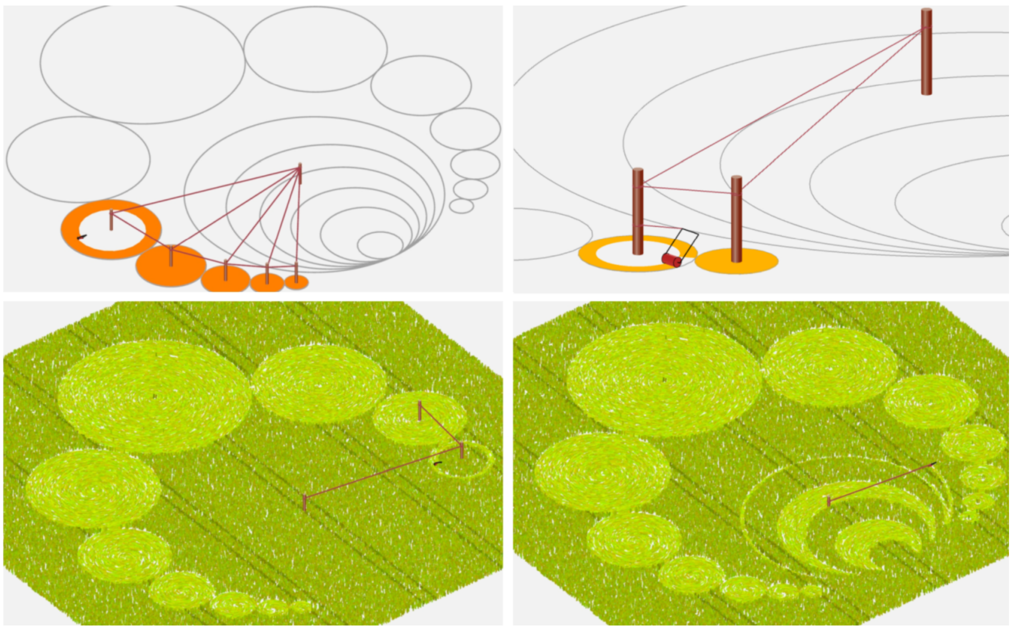

Figure 4: Creation of the complex circle pattern with the help of a few people equipped with pegs, ropes, and a roller.

Artistic disk B-spline shapes by skinning

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We study an alternative shape modeling tool to standard control point based modeling techniques by extending control points to control disks, resulting in an easy and straightforward way of modeling regions or shapes with adjustable thickness. This technique is called disk B-spline curves that are frequently applied in artistic shape modeling applications, such as brushstroke representation, calligraphy, and animation, but they are also suitable for engineering purposes. However, the boundary curves of the described shapes tend to have unintentional self-intersections or cusps, negatively affecting the usability and artistic value of the resulting shape and texture. In this talk, we introduce an iterative algorithm to give a solution to this problematic issue with the help of an approximating circle skinning method, while preserving the advantageous properties of the disk B-spline curves. Our method also results in smoother texturing of the shape around the more sharply curved sections.

MSC 2010: 41A05, 41A10, 65D05, 65D17

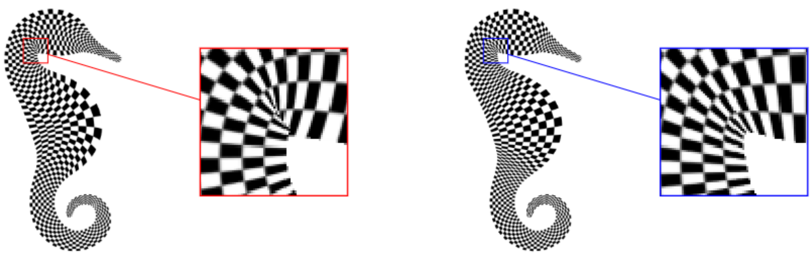

Figure 1: Comparison of textures with the classical DBSC curve (left) and with our method (right).

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Homogeneous ideal of embedded modular curves $X_0(N)$

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A modular curve $X_0(N)$ is a compact Riemann surface defined by group action of congruence subgroup $\Gamma_0(N)$

$$\Gamma_0(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2(\mathbb{Z}) : c \equiv 0 \mod N \right\}$$

of modular group $SL_2(\mathbb{Z})$ by linear fractional transformations

$$z \mapsto \frac{az+b}{cz+d}, \qquad z \in \mathbb{H} \cup \mathbb{R} \cup \{\infty\}$$

on the complex upper-half plane extended by the point at infinity and the real axis. It is known that every Riemman surface can be embedded, injectively mapped, to a projective curve. We observe such maps defined by holomorphic functions on the complex upper-half plane, namely cuspidal modular forms.

The image curve is a projective curve and we look at the graded homogeneous ideal of homogeneous polynomials vanishing on the curve and its minimal generators. When we map to $\mathbb{P}^n$ and $n \geq 2$, then the curve is always given by a system of polynomial equations and the generators of the ideal give the minimal such system. Here we use commutative algebra and homological algebra to study graded modules of homogeneous polynomials vanishing on the curve, and we use the modular forms to compute these polynomials.

Further, we compute the Hilbert polynomial, Betti tables and resolutions of our curves to understand the equations that define it.

Key words: modular curve, modular forms, projective curve, homogeneous ideal

MSC 2020: 11F11, 13F20

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Polynomials on cusp forms for $\Gamma_0(N)$ and triangle of numbers

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In this presentation we compute the bases of homogeneous polynomials of degree d such that they vanish on cuspidal modular forms of even weight $m \geq 2$ that form a basis for $S_m(\Gamma_0(N))$. Among them we find the polynomials that are irreducible. We observe the calculated numbers of such polynomials and arrange them in triangle of numbers for which we present some interesting properties regarding to Pascal and Rascal triangle.

Key words: modular forms, modular curves, triangle of numbers

MSC 2020: 11F11, 13F20, 05E40

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Could a wizard make geometry easier to learn?

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There is considerable interest in integrating digital games into mathematics education, due to their potential to enhance student engagement and make the subject matter more appealing. Effective implementation of game-based learning demands the development of appropriate tools and the establishment of systematic procedures for their utilization. These tools and methods should be created by multidisciplinary teams including professionals in pedagogy, mathematics, game design, and programming.

In this talk, we will present GeomWiz, a gamified geometry quiz developed as part of the GAMMA (GAMe-based learning in MAthematics) project for upper secondary school students, along with two accompanying teaching scenarios.

Throughout the project's duration, teachers implemented GAMMA games and scenarios in the classroom. Both teachers and students provided feedback on their digital game-based learning (DGBL) experience through a short questionnaire. Post-pilot surveys on the GeomWiz game will be analyzed using both qualitative and quantitative methods.

Key words: mathematics education, digital games, digital game-based learning (DGBL), teaching scenario, user experience

MSC 2020: 97G30, 97G40, 97G60

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Leapfrogs recurrence relations for continuant polynomials

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The continuant is an algebraic object defined as a multivariate polynomial. These polynomials are essential to the study of continued fractions and were extensively studied by Euler. In addition, continuant polynomials have implications to Stern-Brocot tree, quivers and some other algebraic, number theoretical and combinatorial objects. In this work we present families of identities for continuants. The proofs are done by means of enumerative arguments.

Key words: continuants, continued fractions, leapfrogs, recurrence relation, polynomial identities

MSC 2020: 05A19, 05A99

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On modular forms constructed from Wronskians

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Joint work with Goran Muić

Let Γ be the Fuchsian group of the first kind. We give in depth study of the divisors of modular forms given by Wronskians of modular forms. For $\Gamma = SL_2(\mathbb{Z})$ we give an example which improves on ([1], Theorem 3.7) at least in a particular but substantial case.

Key words: modular forms, algebraic curves, uniformization theory, Weierstrass points

MSC 2020: 11E70, 22E50

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Exploring students' graph interpretation in the context of mathematics and physics education by eye-tracking

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Reading, interpreting, and constructing graphs are essential skills in both mathematics and physics education. In this presentation, we focus on graphs related to the concept of speed, a notion that is present in both disciplines. We will provide an overview of recent research on the subject and also present the findings of a recent joint study that investigated two problems: the stone problem and the well-known Racing car problem. The study was designed to address students' difficulties in interpreting motion graphs and was analysed by means of eye-tracking technology.

Key words: Mathematics education, Graph interpretation, Racing car problem

MSC 2020: 97D70, 97G20, 97M10

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Twice punctured Euclidean and hyperbolic 3-manifolds and "quantum dots"

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My about 40 years old paper [1] had got a surprising actuality in the Chemistry Nobel Prize 2023 award shared by the three Laureates, Alexey Ekimov, Luis E. Brus and Moungi G. Bawendi.

Of course, the present author of that paper could not guess that time the actuality and importance that was an incidental consequence of my erroneous paper [2], intended to construct an infinite series of non-orientable compact hyperbolic manifolds, as a polyhedral tiling series in the Bolyai-Lobachevsky hyperbolic space H^3.

Fortunately, I found and corrected the mistakes soon. Namely, those constructions were not manifolds, because of the two fixed point orbits as punctures, where points reflections (central inversions) occur in the symmetry group of the tricky polyhedral tilings. But these singular points, as "quantum dots", e.g. for copper and chlorine ions, respectively, in glass (silicon) fluid, cause light effects (by "electron jumping-leaping") whose colours might depend on the sizes of crystal particles.

That means, the mistake was much more interesting than the original intention that can be reached easily later!

Key words: non-orientable compact hyperbolic manifolds, polyhedral tiling series, "quantum dots"

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Spatial ability in vocational training

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Spatial ability is essential for an expert to be successful in several disciplines.

According to previous studies, spatial intelligence and spatial abilities are predictors of success in technical education and have a high importance in engineering education, computer graphics, architecture, arts and cartography. In addition, there are correlations between spatial intelligence and performance of Science, Technology, Engineering and Mathematics (STEM); spatial ability is needed in vocational training and in higher education.

Spatial intelligence has an important role in learning and teaching of engineering studies. The skill of mental manipulation of objects is particularly important for engineering students.

This report investigated the spatial visualization skills and task types of students at the University of Debrecen in Vocational Teacher Training in four specializations.

Key words: descriptive geometry, engineering, vocational education, intelligence

MSC 2020: 97G80, 97M50, 97B30, 97C40

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Side-side-Angle triangle congruence axiom and the complete quadrilaterals

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Triangle congruences play a significant role in Euclidean geometry. Some of them are axioms in almost all geometric structures. Currently, the triangle congruence axioms have attracted the interest of a number of researchers, such as Donnelly [3, 4], who examined their role in absolute geometry. The so-called Side-side-Angle (SsA) triangle congruence is the most complex of the triangle congruences. Some researchers disagree that it is a fundamental triangle congruence axiom.

SsA. Two triangles are congruent if and only if two pairs of corresponding sides and the angles opposite the longer sides are equal.

We understand that two triangles are not necessarily congruent if two pairs of corresponding sides and the angles opposite the shorter sides are equal. In this case, two non-congruent triangles are possible under these conditions. It led us to examine the connections between these two triangles satisfying the following condition.

Condition sSA. Two triangles hold the condition sSA if their two pairs of corresponding sides and the angles opposite the shorter sides are equal.

We fix and superimpose the shorter sides of two triangles under the condition sSA, and we demonstrate that the locus of the intersection points of the side lines is a hyperbola.

Theorem 1 *[1] Let A and B be two fixed points. If the triangles ABC and ABC', with Condition sSA have their corresponding shorter sides AB, and the common lengths of their corresponding longer sides AC and AC', respectively, are fixed, then the locus of the intersection points of the corresponding lines of sides is a hyperbola.*

Moreover, we show that the locus of the intersection points of the side lines is a hyperbola if the angles opposite the shorter sides are also fixed.

Theorem 2 *[2] Let A and B be two fixed points. If the triangles ABC and ABC', with condition sSA have their corresponding shorter sides AB, and the common angles $\gamma = \gamma'$ opposite the shorter sides are fixed, then the locus of the intersection points of the corresponding lines of other sides is an orthogonal (normal) hyperbola.*

We also show some generalized results not only in Euclidean, but also in projective plane.

Key words: triangle congruence, side-side-angle axiom, complete quadrilateral, conic section

MSC 2020: 51N20, 51N05, 51N35

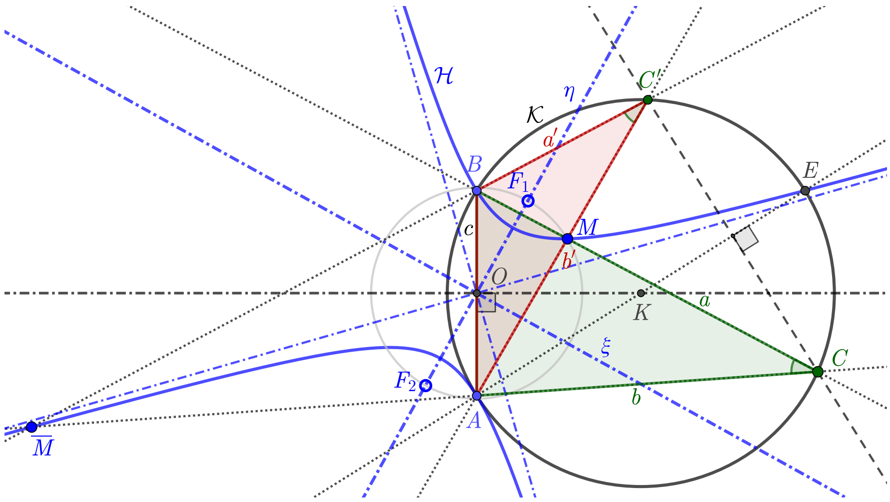

Figure 2: Locus of the point M Axis of hyperbola $\mathcal{H}$

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Universal porisms and Yff conics

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Among the huge variety of tritangent and cricumscribed conics of a triangle, the Yff conics (cf. [3, 7]) play an outstanding role. Many of them do not depend on Euclidean notions, *e.g.*, the side lengths of the triangle. Especially the Yff circumellipse $\mathcal{M}$ and the Yff inellipse with the unit point of the projective frame as the common Brianchon point (see [4]) allow for a poristic family of triangles interscribed between them (see, *e.g.*, [2]). It turns out that this poristic family can be parametrized by means of polynomial functions (provided homogeneous coordinates are used). Thus, we are able to give explicit examples of poristic triangle families in projective planes over arbitrary (finite) fields (see [1, 6]) of any characteristic which makes these porisms also a notion of *Universal Geometry* (in the sense of [8]). Moreover, from the conics in the exponential pencil spanned by $\mathcal{M}$ and $\mathcal{N}$ (as defined in [5]), we can construct a sequence of nested rational triangle porisms.

Key words: porism, inellipse, circumellipse, triangle, rational porism, rational parametrization, finite field, finite projective plane

MSC 2020: 14Pxx, 51N15, 51N35

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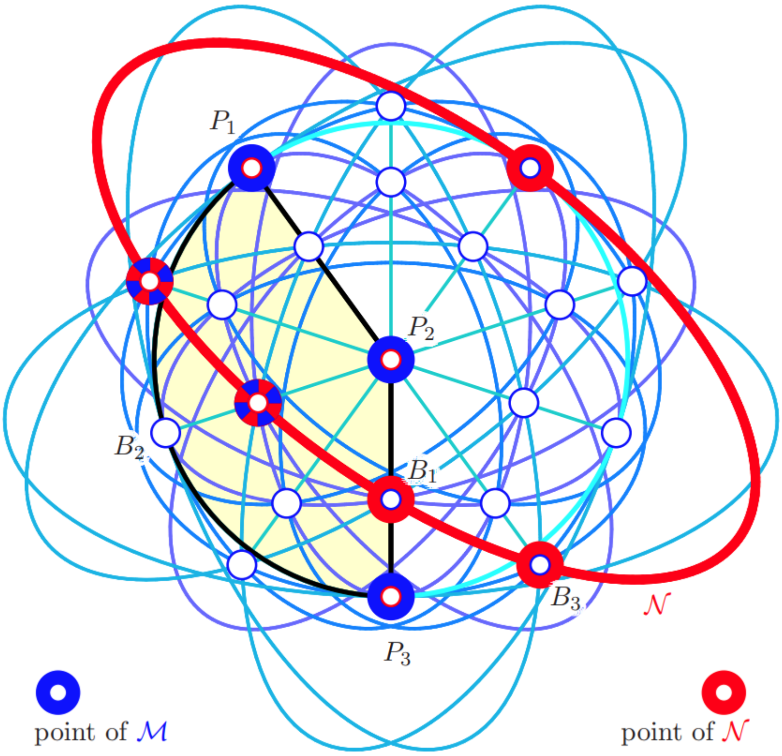

Figure 3: In $\mathbb{P}^2(\mathrm{GF}(4))$, the "circumconic" $\mathcal{M}$ is regular, while the "inconic" $\mathcal{N}$ consists of five collinear points. Further, $\mathcal{M}$ and $\mathcal{N}$ share two points playing the role of the degenerate triangles in the poristic family. The yellow triangle corresponds to the parameter value $u : v = 1 : 0$.

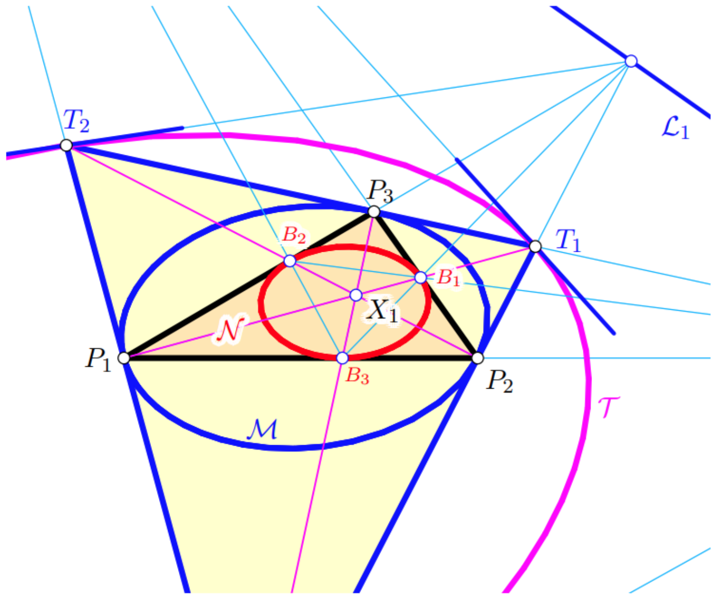

Figure 4: Iterating the Yff porism yields infinitely many universal porisms. All conics share the Brianchon point X_1 to which the tangent triangles are perspective.

AI guided auxiliary constructions

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Geometric problem solving is a challenging field for Artificial Intelligence (AI). Large Language Models (LLM) have greatly improved in recent years, and therefore, combining LLM, Algebraic Reasoning (AR) and a Deductive Database (DD) was the next step in hopes of getting a stronger and more robust AI to solve geometric problems. In this study, we look at recent advancements, specifically the AlphaGeometry Project to analyze an AI on its capabilities to find answers. We explore the approach of the AlphaGeometry system, which tries to find the solution by adding auxilary constructions under guidance of a neural network. Building a useful neural network, which can suggest the next auxiliary construction needs a big and versatile set of training- and testdata. Therefore finding ways to synthesize valuable training- and testdata aids in building a strong AI. By using different training- and testdata along with an extended DD to expand the range and efficiency of the Model, we are aiming to solve geometric problems with higher complexity.

Key words: AlphaGeometry, Automated Geometric Theorem Prover, Algebraic Reasoning, Deductive Database, Large Language Model

MSC 2020: 51M04, 68T07, 68T20

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Canal surfaces containing four straight lines

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A canal surface is the envelope of spheres with variable radius and centers traversing a spatial curve. Each sphere contacts the envelope along a circle called characteristic. We are interested in canal surfaces containing lines that are not the limits of characteristics when the radius of enveloping spheres tends to infinity. There are trivial cases of canal surfaces through infinitely many lines, the right cylinders and cones and the one-sheeted hyperboloids of revolution. The only nontrivial case of a canal surface which contains four (non-characteristic) straight lines is related to a Plücker conoid. The four given lines must be concyclic, i.e., they intersect each tangent plane of the conoid in four points lying on a circle [1]. We are going to analyse these particular canal surfaces, which in general have singularities in form of cuspidal edges. As limiting cases, also parabolic Dupin ring cyclides are included.

Key words: canal surface, spine curve, Plücker's conoid, concyclic generators

MSC 2020: 51N20, 53A05, 51N35

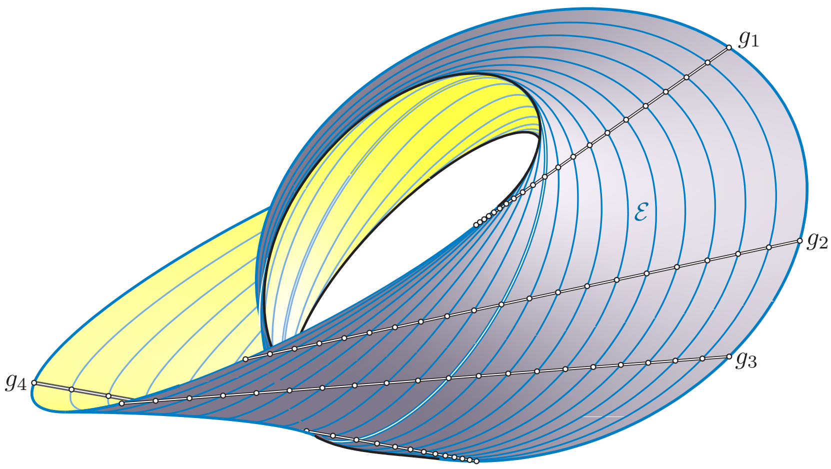

Figure 5: A symmetric example of a canal surface $\mathcal{E}$ through four lines $g_1, \ldots, g_4$. The black curves are edges of regression; the double line marks the smallest circle.

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Modular timber architecture

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Students at Graz University of Technology have designed and developed a spatial sculpture in their Master's studio at the Institute of Architecture and Media that can be made from just two slightly different small flat wooden panels. Each of the two flat parts has a dimension of approx. 20cm x 50cm. The basic shape of the sculpture follows cylinders of revolution with tangentially attached flat parts. When assembled, the modular parts follow geometrically the circles and the generators of the cylinders. All parts could be produced very easily on a 3-axis CNC milling machine, and no special logistics had to be applied to differentiate the parts.

This project therefore follows the philosophy of creating space, form and complexity through simplicity. The aesthetics of the overall shape is achieved by combining the different cylinder parts and their boundaries, some of which are helical. A relatively simple geometry thus creates a beautiful work of spatial art.

In the presentation we will go into the individual steps of the creation and above all the difficulties and mistakes that had to be solved until the clear approach of the construction guided by the geometry had emerged.

Key words: architectural geometry, modularity

MSC 2020: 00A06

On the isoptic point of the non-cyclic quadrangle in the isotropic plane

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We study some properties of the non-cyclic quadrangle $ABCD$ in the isotropic plane related to its isoptic point. The motivation was found in our earlier research, where we investigated the isoptic point of the complete quadrangle in the Euclidean plane. In the isotropic plane, we put the non-cyclic quadrangle in the standard position, which enables us to prove its properties using simple analytical method. In the standard position, the special hyperbola $xy = 1$ is circumscribed to the quadrangle. We give several results related to the isoptic point of the non-cyclic quadrangle. The isoptic point T is the inverse image of points A', B', C', D' with respect to circumcircles of BCD, ACD, ABD, ACD, respectively, where A', B', C', D' are isogonal points of the vertices A, B, C, D with respect to triangles BCD, ACD, ABD, ACD. The circumircles are seen from T under the equal angles.

Key words: isotropic plane, non-cyclic quadrangle, diagonal points, isoptic point

MSC 2020: 51N25

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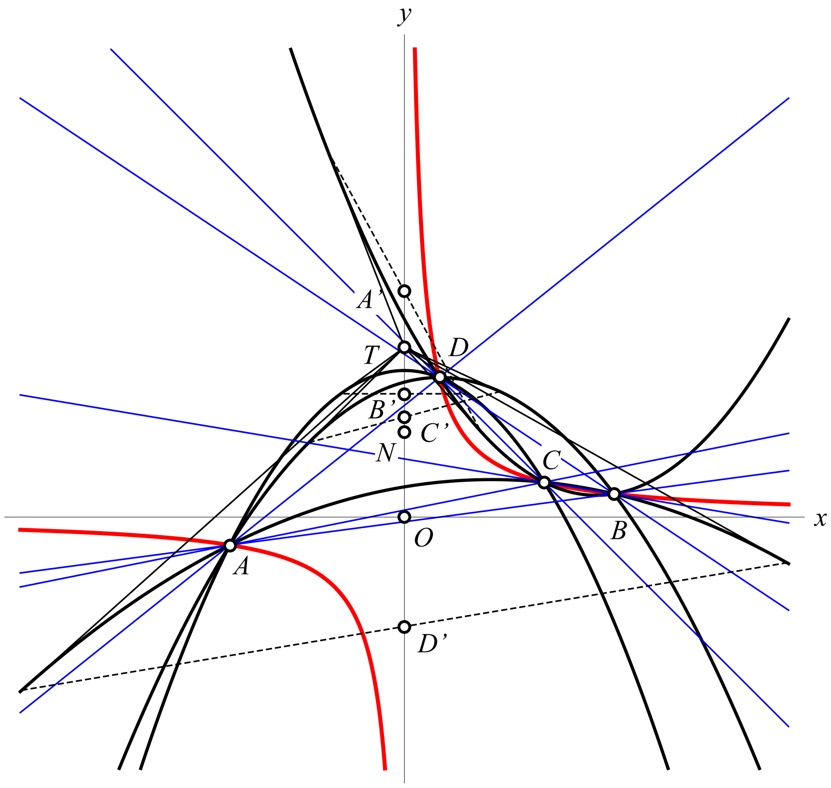

Figure 1: The non-cyclic quadrangle $ABCD$ and its isoptic point T

Bijective digital rotation on hexagonal grid

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Digital rotations are frequently used transformations in image processing [1, 2, 3]. During these transformations, two well known errors can be faced. The first case is when, after the transformation, the images of two different pixels will have the same coordinates (error of type-1) and the second case is when, after the rotation, the previously neighbouring pixels will no longer be neighbours (error of type-2).

This presentation provides a novel algorithm based on digital circles for rotation. This method always gives a bijective mapping from the original picture to the rotated one showing that no error of type-1 may occur. This method is independent of the shape of the grid.

Our research focuses on hexagonal grid. We define the possible neighbour configurations for “wide” contours which can preserve the connectedness of the images. Therefore during the rotations the topology remain the same as the original. We compare the measure of its errors with the naïve approach and the shear-based rotation of Eric Andres [4, 5].

Key words: digital rotations, hexagonal grid, digital circles, digital continuity, bijective transformation

MSC 2010: 52C20, 62H35, 68U10, 94A08

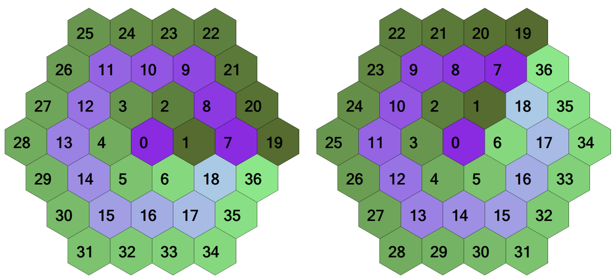

Figure 1: Hexels of a digital image before and after rotation.

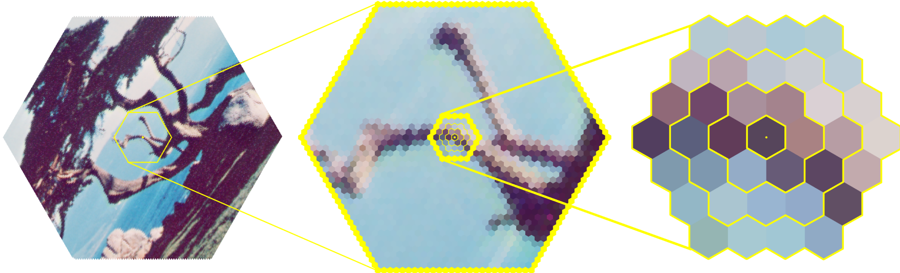

Figure 2: Digital circles of the image.

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Symmetry – the second service

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Symmetry is a vast subject, significant in art and nature. Mathematics – geometry lies at its roots and it would be hard to find a better one on which to demonstrate the working of the mathematical intellect.

Hermann Weyl

Symmetry as a general concept of study might appear in two different forms (but mixing both approaches is often possible, too):

I. Model of the system is given, encoded in action, mathematical formula, or a set of equations describing some phenomena. Then spotting a symmetry of the model provides insight into the character of the solutions, and often helps to find them by:

   - identifying the appropriate coordinate system in which the equations are easier;
   - providing an algorithm to make new solutions from those already obtained;
   - giving a hint that something is still missing to have the complete set.

II. Symmetry is given as an assumption, or is, for some reason, expected to be present. Then it can be used as a lead to build the model by:

   - excluding possibilities without the required symmetry, thus narrowing down the possible choice;
   - searching for solutions foreseen due to symmetric property that might be not directly obvious;
   - using classification keys, knowing that a member of a family tells also something about the others.

One of the presented examples is the concept of homological mirror symmetry used in quantum physics to the Calabi-Yau manifolds, whose shapes are mathematically described by an array of Hodge numbers. This approach helps to find solutions to the famously hard problem: to find numbers of rational curves of a given degree (representing the number of their loops around) for the simplest Calabi-Yau space, the Fermat quintic treefold.

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Stellae octangulae in motion

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It is known that two regular tetrahedra T_1, T_2 forming a stella octangula allow a motion of T_2 relative to T_1 such that all edges of T_2 slide along all edges of T_1, (c.f. Stachel, 1988, p. 65–75). We can extend this property for stellae octangulae consisting of two symmetric tetrahedra of general form. It turns out that, besides some special one-parametric motions, there exists, within limits, also a two-parametric set of such motions.

Key words: Tetrahedron, Stella Octangula, Euclidean motion

MSC 2020: 51N20, 51N30

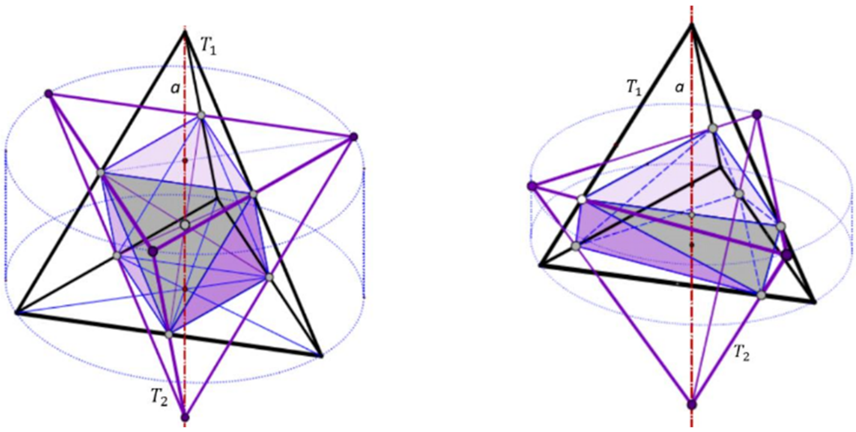

Figure 1: Stella Octangula consisting of two tetrahedra T_1 and T_2 in extremal start position (left), and in an intermediate position (right). At the start position, the intersection $T_1 \cap T_2$ is an octahedral antiprism with a regular face triangle at the top and at the bottom and six congruent isoceles triangles in between (left). At the intermediate position, $T_1 \cap T_2$ is still is antiprism with regular top and bottom, but with irregular face triangles between top and bottom (right).

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Optimal transport for mesh repair

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In photogrammetry-based 3D-Scanning, a multitude of photographs of an object are taken and three-dimensional structures are estimated via Structure from Motion. These 3D-Data can then be triangulized to create 3D-Meshes. However, the results usually contain artefacts or some parts may be missing. Repairing a mesh is a cumbersome process when done by hand and requires speciality software when done automatically. This, for example includes the smoothing of surfaces, outlier removal, and closing of holes.

Optimal transport has enjoyed a variety of applications in recent years. In this study, we want to concentrate on 3D-point clouds as data-source. Wasserstein barycenters offer a way of transforming features from one 3D point cloud to another. We test whether this can be used to repair meshes or parts thereof as they are generated in photogrammetry-based 3D-scanning processes. Automatically completing the scan with parts of the same object which in turn can be duplicated and pasted would make mesh repair a simpler process. To make this user-friendly special attention should be paid to runtime of the algorithm and ease of use. Related approaches have already shown that Wasserstein barycenters can be used to transform features, this can be seen in Figure 1. We adapt this algorithm to be tested for the repair of partial meshes resulting from 3D Scans.

Key words: photogrammetry, 3Dscan, mesh repair, optimal transport, Wasserstein barycenter

MSC 2020: 65D18, 60A10, 65K10, 49Q20, 49M29

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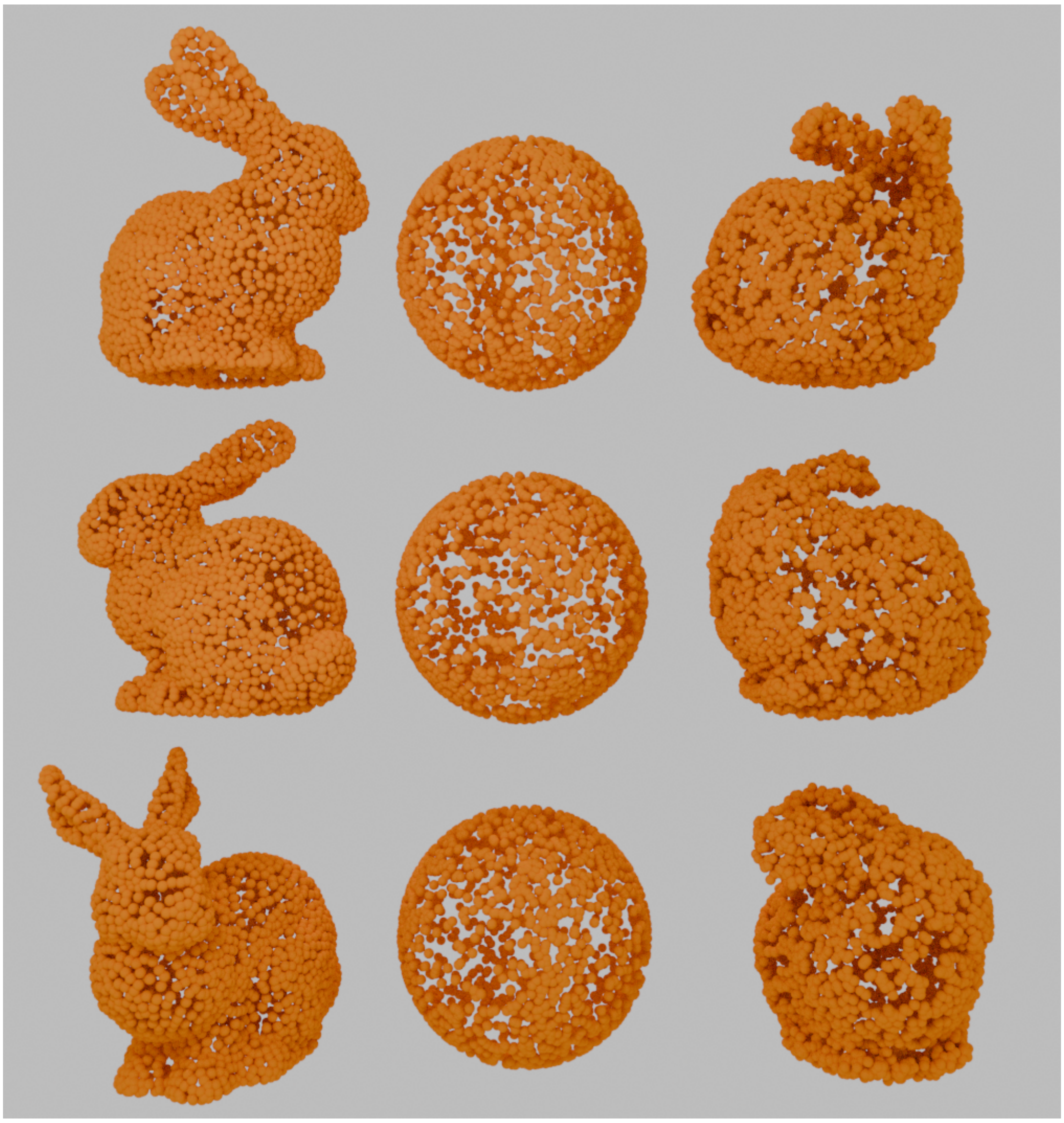

Figure 1: This figure shows the point clouds of the Stanford Bunny, a randomly sampled surface of a sphere and the Wasserstein barycenter of the two, rotated around their axes. The bunny becomes inflated and round. A similar approach might be used to transfer other features between point clouds.

Posters

Generalization of Steiner's theorem

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Analyzing Steiner's constructions, we conclude that we can reach the deltoid by observing a pair of lines a and b with the intersection S on the circle k instead of the triangle inscribed in the circle k, where we establish a 1-2-correspondence between two ranges of points (a) and (b) whose product will be the required envelope. It is also interesting to show what happens in the case when the intersection S is outside or inside the circle. And what happens in the case when the aforementioned 1-2-correspondence is organized in a different way.

Key words: Steiner's constructions, detoid, astroid

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Generalized spherical helices in 3-dimensional Minkowski space

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A regular space curve is called a generalized helix if its tangent vectors make a constant angle with a fixed straight line, called the axis of a generalized helix. Of special interest are spherical generalized helices due to their interesting geometric properties [3]. We present our analysis of generalized spherical helices in 3-dimensional Minkowski space, i.e. of generalized helices lying on a pseudosphere $S_1^2(p,q)$ and in a hyperbolic plane $H_2(p,q)$ [1] and extend this notion by taking into account curves lying on a lightlike cone as well, [2]. We studied their projections on planes orthogonal to their axis which appear as Euclidean epi-, hypo- and paracycloids or their Minkowski counterparts.

Key words: Lorentz-Minkowski space, generalized helix, spherical curve, cycloidal curve

MSC 2020: 53A35, 53B30

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On isogonality in a complete quadrangle

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Joint work with Vladimir Volenec

Numerous authors have studied a complete quadrangle in the Euclidean plane and proved its properties in different ways. Our goal was to collect as many facts as possible about the complete quadrangle and prove them all by a single method. We put the complete quadrangle $ABCD$ into such a coordinate system that its circumscribed hyperbola $\mathcal{H}$ is rectangular, and its vertices have coordinates $A = \left(a, \frac{1}{a}\right)$, $B = \left(b, \frac{1}{b}\right)$, $C = \left(c, \frac{1}{c}\right)$, $D = \left(d, \frac{1}{d}\right)$, which simplify analytical computations. We use this method to prove some already published theorems and to derive some new and original ones.

On this poster we focus on the facts related to the diagonal triangle of the quadrangle $ABCD$, as well as on the isogonality with respect to the triangles BCD, ACD, ABD and ABC.

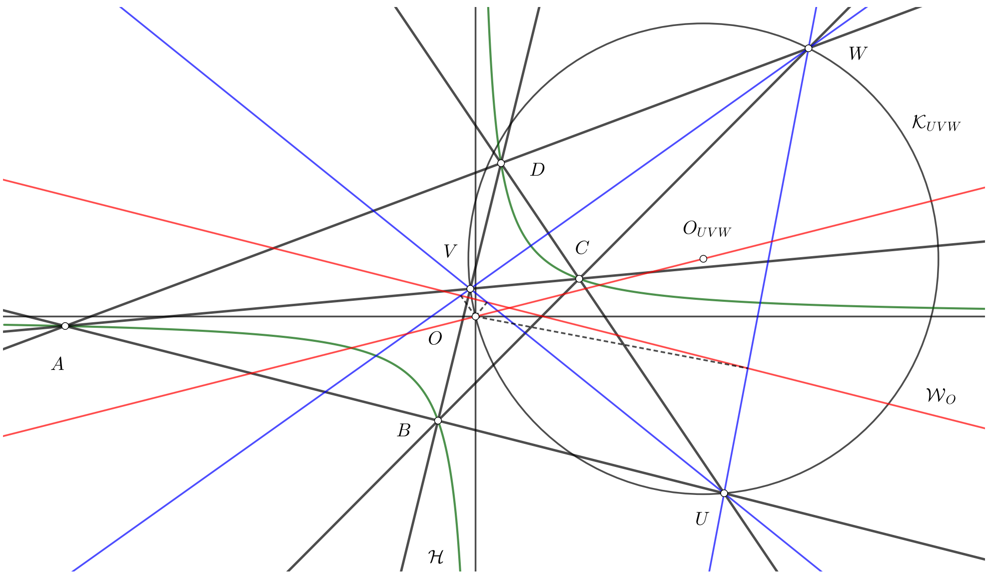

Figure 1: The Wallace's line $\mathcal{W}_O$ of the center O of the quadrangle $ABCD$ with respect to the diagonal triangle UVW and the line OO_{UVW} form equal angles with the asymptotes of $\mathcal{H}$.

Key words: complete quadrangle, diagonal triangle, isogonality

MSC 2020: 51N25, 51N20

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The application of parametric modelling

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The poster contains various examples of the application of parametric modelling and differential geometry. Examples of student work will be illustrated with the pictures of the program and corresponding models. The requirement for the students was to complete three assigned problems of the open type to encourage problem solving and imagination. The student models were made by the use of *Rhinoceros3D* and *Grasshopper* within the elective course "Geometry in Civil Engineering" at the Faculty of Civil Engineering, University of Zagreb.

Key words: Grasshopper, parametric modeling, minimal surfaces

MSC 2020: 65D17, 53A04, 53A10

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Weberian surfaces with four foci and four directorial planes

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Weberian surfaces, defined as geometric loci with a constant sum of scaled distances from either foci, directorial lines, or planes, are a fruitful basis for generating geometric shapes. Herewith we present novel Weberian surfaces as geometric loci with a constant sum of scaled distances from four foci and four directorial planes, defined by the following equation:

$$\sum_{i=1}^{4} a_i R_i + \sum_{j=1}^{4} b_j h_j = S,$$

where R_i is the distance of the point of the Weberian surface from the i-th focus F_i and h_j is the distance of the point of the Weberian surface from the j-th directorial plane H_j (Figure 1a), while the coefficients a_i, b_j and S are real numbers ($i = 1, \ldots, 4$, $j = 1, \ldots, 4$). We can vary the form of such surfaces by varying the corresponding coefficients; some examples are given in Figure 1 (b, c, d).

These algebraic surfaces, either in whole or in part, may be used as patterns in the design of architectural spaces, [2–5].

Key words: focus, directorial plane/directrix plane, locus of points, Weberian coefficient, algebraic 3D surface

MSC 2020: 53A05, 51N20

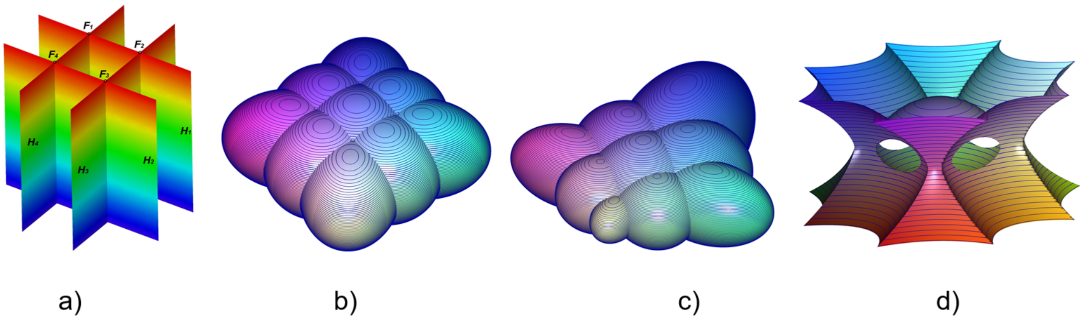

Figure 1.

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